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A Theoretical and Experimental Study on Cake Filtration with Sedimentation

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Abstract–Sedimentation occurs in almost all cake filtration. To study the role of sedimentation during filtration, filtration-permeations were carried out for suspensions of various concentrations. The average specific cake resistances calculated by the traditional method using the filtration data and initial concentration of suspension give different values according to the suspension concentrations. But the average specific cake resistances from the permeation period show almost the same values in spite of the various suspension concentrations. To exclude the influence of sedimentation, a complete sedimentation was performed before beginning filtration and then filtration-permeation of the sediment was performed. The average specific cake resistance from the filtration period calculated with the mass fraction of sediment and that from the permeation period coincide very well. These values also coincide well with the former average specific resistances during permeation within the experimental error limits. It can be concluded that the average specific cake resistances by permeation operation give almost the same values for various concentrations of suspension. A new definition of a cake and a new concept of the filter medium R_m are proposed based on the analysis of experimental results.

Key words : Filtration, Cake Filtration, Sedimentation, Filtration-Permeation

INTRODUCTION

Sperry [1917], in the pioneering study on filtration theory, worried about sedimentation during filtration, since it would interfere with the accurate measurement of the average specific cake resistance. Although many investigators studying filtration phenomena have certainly thought about this subject, only a few studies have been published because of the theoretical and experimental difficulties for analyzing the phenomenon. The first approach to this subject was by Bockstal et al. [1985], who assumed that the sediment upon a filter cake during filtration was a filter cake. This assumption will be one of the most important subjects of our study. Tiller et al. [1990] presented the process of filtration with sedimentation using the "computerized axial tomographic scanner" at the 5th world filtration congress, but a theoretical explanation of the phenomena for filtration with sedimentation was not available. Although Tiller et al. [1995] gave a theoretical explanation to the above phenomena, they made no distinction between the sediment and the filter cake as did Bockstal et al. [1985].

The author [1990] explained the cause of the difference between the two average specific cake resistances, i.e., one measured during filtration accompanied with sedimentation and the other measured during permeation which did not have the sedimentation. This was not possible by Tiller et al. [1990] at that time.

The purposes of this study are to illustrate the difference in phenomena between filtration with and without sedimentation, to theorize about filtration with sedimentation, and to prove this theory by the filtration-permeation technique proposed by the author [1986].

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THEORETICAL STUDY

1. Analysis of the Filtration Process Accompanying Sedimentation

Almost of all filtrations are accompanied more or less with sedimentation. The constitution of the filtration process accompanying sedimentation in this study is assumed as below.

At the beginning period of filtration the flow rate of filtrate is fast because the thickness of cake at the period is thin, but even during this period the sedimentation also progresses. A solid blanket, which is the interface between suspension and supernatant, begins to form at this period for relatively thick suspensions. The formation of the solid blanket during filtration is an apparent evidence of the existence of sedimentation, but this evidence had been ignored by almost all of the researchers. For filtration of suspensions which are not so thick as to form a solid blanket, clear supernatant is usually observed at the upper part of the suspension being filtered. This phenomenon is also an obvious evidence of the existence of sedimentation during filtration.

In Fig. 1, the phenomena of filtration with sedimentation explained above are shown. Fig. 1(a) represents the moment at beginning filtration, and Fig. 1(b) shows the state that a period of filtration passed. The difference of water level between two figures is due to the filtrate which has gone out of the filter cell through the filter medium. If there were no sedimentation, the solid particles which had been in the filtrate would only form the filter cake in Fig. 1(b). In our case, sedimentation occurs with filtration ; thus the supernatant resulting from

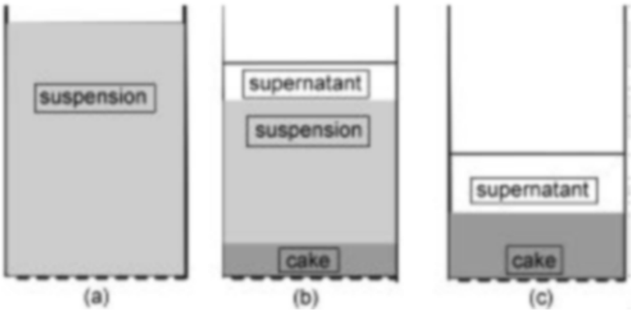


Fig. 1. Schematic diagram of the process of filtration with sedimentation.

sedimentation exists in the upper part of the cell, and below it lies the suspension for filtration. The portion beneath the suspension is the cake formed by filtration, and then filter medium. Until now, the only difference between the filtrations with and without sedimentation is the existence of supernatant.

Darcy's equation is normally used in filtration without sedimentation as below:

$$\frac{dV}{dt}=\frac{\Delta P}{\mu(\alpha_{av}W+R_m)} \tag{1}$$

Here, V is the filtrate volume per unit filter area (m^3/m^2), t is the filtration time, ΔP is the filtration pressure, μ is the viscosity of filtrate, α_{av} is the average specific cake resistance, W is the mass of cake solid per unit filter area, and R_m is the resistance of filter medium.

Taking the inverse of Eq. (1) leads to

$$\frac{dt}{dV}=\frac{\mu\alpha_{av}}{\Delta P}W+\frac{\mu}{\Delta P}R_m \tag{2}$$

This equation is used for analysing the experimental method "filtration-permeation" developed by the author [1986].

Bockstal et al. [1985] and Tiller et al. [1995] assumed that the cake mass during filtration accompanying sedimentation increases not only with the progress of filtration but also with the amount of sediment. This means that the sediment piles up onto the filter cake and, this is very important, the sediment acts as a cake from that moment. It was also assumed that the amount of supplementary cake by sedimentation could be calculated by sedimentation velocity and the concentration of suspension. This notion means that all of the particles piling on a cake change into a cake immediately. We shall discuss this carefully.

The above notion was not adopted in this study. The author assumed that Ruth's equation, which has been used for filtration without sedimentation, can also be employed to filtration with sedimentation.

$$\frac{dV}{dt}=\frac{\Delta P}{\mu\left(\alpha_{av}\frac{\rho S}{1-S/S_c}V+R_m\right)} \tag{3}$$

Here, ρ is the density of filtrate, S is the mass fraction of solids in suspension entering the cake, and S_c is the mass fraction of solids in the cake. This equation is also changed as below to analyze the experimental data.

$$\frac{dt}{dV}=\frac{\mu\alpha_{av}}{\Delta P}\left(\frac{\rho S}{1-S/S_c}\right)V+\frac{\mu}{\Delta P}R_m \tag{4}$$

Bockstal et al. [1985], Tiller et al. [1995] and almost of all other investigators have adopted the initial mass fraction of solids in suspension as the value of S and assumed that the value remains constant during all the procedure of filtration. But the exact meaning of S is the mass fraction of solids in suspension **entering onto a cake**, and is not the average mass fraction of solids in suspension at the moment of beginning filtration. When filtration is accompanied by sedimentation, the mass fraction of suspension entering into a cake cannot be maintained at the same initial value. We can prove it with the existence of a solid blanket which separate the suspension from supernatant.

In spite of sedimentation, we temporarily assume that the suspension with initial value of S enters into a cake during all of the filtration procedure. The suspension which has already entered into the cake in Fig. 1(b) has the initial value of S. Then the mass fraction of solids in the sum of both supernatant and suspension above the cake in Fig. 1(b) must have the same initial value of S. As the mass fraction of solids in supernatant is very small, that in suspension must have a larger value than the initial S. So if the suspension of larger value of S is introduced in the cake, the above assumption that the suspension of initial S enters into a cake during all of the filtration procedure is not adequate. As this logic is true at any moment of filtration, the application of constant initial value of S to Eqs. (3) or (4) must be reexamined.

In this study, it is assumed that the only difference between the filtrations with and without sedimentation is the mass fraction of solids in suspension S due to sedimentation as mentioned above. To have a clear notion about our new concept, it is compared with the former theory.

As already mentioned, the notion of Bockstal et al. [1985] and Tiller et al. [1995] for the filtration with sedimentation is that the sedimented particles are considered as a part of filter cake. Thus the mass of cake per unit filter area, W, was calculated as the sum of the amount of cake formed by filtration, i.e. $\rho SV/(1-S/S_c)$, and the amount of sediment, $\rho Sv_st/(1-S/S_c)$. Here, v_s is the sedimentation velocity. If this would be true the mass fraction of solid in sediment and in cake must be equal. The mass fraction of solids in sediment is generally smaller than that in filter cake; this fact will be shown later with experimental results. Thus the simple addition of sediment mass and cake mass to calculate W cannot be accepted by the author. Furthermore, the mass of sediment, by $\rho Sv_st/(1-S/S_c)$ could not be exactly as the solid mass fraction of cake, S_c, and thus cannot be correct. So the author does not agree with the notion that sediment is cake.

The mass of cake W in this study is calculated simply as $\rho SV/(1-S/S_c)$ by Ruth's equation, and the mass fraction of solids in cake, S_c, does not change by sedimentation throughout filtration as has been assumed in conventional cake filtration. The only modification from conventional filtration in this study for describing filtration with sedimentation is that the mass fraction of solids in suspension, S, changes with sedimentation.

2. The Sediment and the Cake: Filtration-Permeation of Sediment

The filtration of sediment, which was developed by the author, was performed in order to prove the difference between sediment and cake. At first, the suspension was put in a filter cell and sedimented without filtration as Fig. 2(a). Then the state Fig. 2(b), when the sedimentation was complete and the sediment piled up on the filter media, was attained. The main difference between the two states is the movement of the solid blanket. At the state of Fig. 2(b), there is little change in the height of the solid blanket. This sediment is cake by the assumption of Bockstal et al. [1985] and Tiller et al. [1995], but it is not adopted in this study.

Filtration was started at this state of Fig. 2(b) and the procedure was as Fig. 2(c). From the upper part there was supernatant, sediment and filter cake. The sediment changed into a cake, i.e., the material being filtered was not the suspension but the sediment. In Fig. 2(c), the sediment and cake are separated with a line, but the interface was not observed visually. But a decrease of the solid height was observed with the increment of filtrate volume. The height of the solid bed is the sum of the sediment and cake height. The solid concentration, i.e., the mass fraction of solids, of initial suspension has no connection with this filtration procedure. In this case the value of S in Eq. (3) is the mass fraction of solids **in sediment** which is much greater than that in initial suspension.

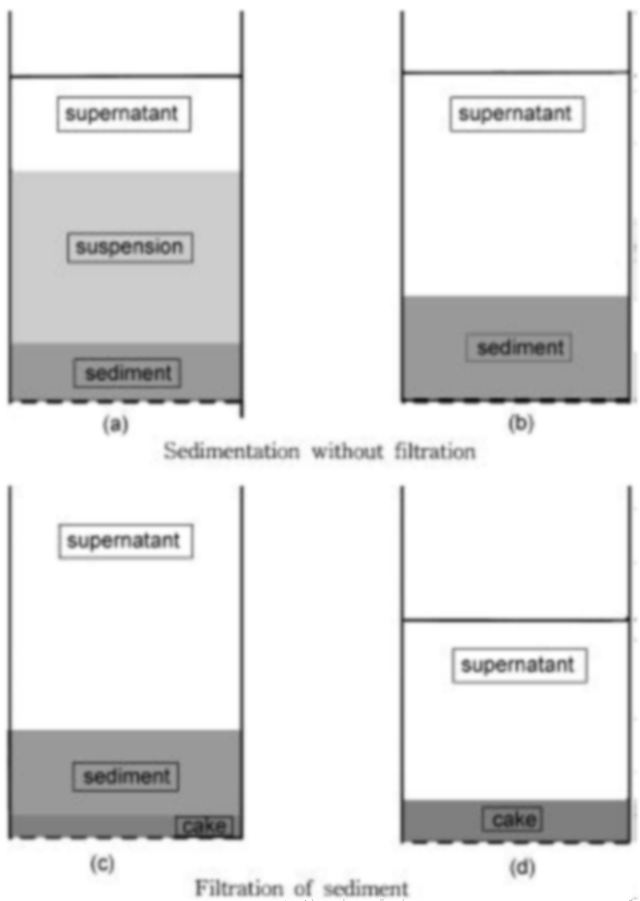


Fig. 2. Filtration of sediment-the concept of sediment and cake.

When all the sediment had changed into cake, the height of solids bed did not decrease any more and maintained the same value as shown in Fig. 2(d). The supernatant passed through the pre-formed cake, which sustained the same mass during this period. The flow rate was constant and obeyed Darcy's equation, Eq. (1).

In conventional filtration, the cake mass in Eq. (1) increases from zero to a certain value, so the flow rate dV/dt decreases continuously. But the flow of supernatant through a completely formed cake, which has constant W, shows constant velocity. The author [Yim and Ben Aim, 1986] presented an experimental technique about filtration and after a permeation of water through the preformed cake for the first time, and termed this experimental method as filtration-permeation. The permeation procedure can be prolonged as long as we want by adding particle removed water to the filter cell.

In this experiment, the water was not added and all of the supernatant passed finally through the cake. When the top surface of the supernatant, i.e., the interface between supernatant and air, touched the cake surface, the expression of the cake was started. The expression process can also be described by Eq. (1) according to the theory developed by the author [1996, 1997].

The aim of 'filtration of sediment' mentioned above, is for verifying the difference between the sediment and the cake. The new concept proposed here is useful for the filtration of wastewater sludge which has been thickened by long time sedimentation.

EXPERIMENTAL

A Büchner funnel was modified as below for more accurate experimental results. Many of the filtration cells have enlarged their cell area about one fifth of the height from the filter medium to put in much suspension. Sediment accumulates at the upper part of the enlargement, so we used straight filter cell to prevent this effect.

Pressure fluctuation is usually generated from an aspirator or a vacuum pump, and it may change the properties of the filter cake. To prevent pressure fluctuation, an air tank of 40 liters was installed between the aspirator and the filtration apparatus. Before filtration, the air tank was depressurized by aspirator to the experimental pressure with the valve between air tank and filter cell being closed. When filtration pressure was attained, the valve between the aspirator and the air tank was closed, and then the aspirator was stopped. Then filtration was begun with opening the valve between filtration apparatus and air tank. The whole process of filtration was performed by the vacuum in the air tank. The maximum filtrate volume at the end was 400 cm^3 and the volume of the air tank was 40 liters, so the maximum pressure variation caused by the volume difference through filtration was less than 1%. As filtration was conducted only by the vacuum of the air tank, leakage at any point of the filtration apparatus was critical to the experimental result. The outside of the connecting part between the filter cell and the support was covered with a tight rubber band to prevent leakage.

Calcium carbonate was used as a raw material. The portion which passed through the 35 mesh sieve was stored at 105 °C. It was cooled in a desiccator for 30 min and then the weight was measured to prevent error due to humidity in particles. The density of calcium carbonate measured by pycnometer was 2570 ± 10 kg/m^3.

The filter media of Toyo Advantec 5C and 0.45 μm cellulose medium of Microfiltration System Co. were used. Filtration characteristics could be changed with filter media [Yim et al., 1997], but the difference was not large in this experiment.

The filtration-permeation method developed by the author [1986] was used for most of the experiments. For permeation with this method, water filtered by 0.2 μm filter (particle-eliminated water) was used. In the experiment, particle-eliminated water was carefully added when about 80 % of suspension in the cell had been filtered. The flow rate, which does not change during permeation, was measured. The average specific cake resistances were calculated by the permeating velocity and Darcy's equation, namely Eq. (2).

RESULTS AND DISCUSSION

1. The Filtration with Sedimentation for Dilute Suspensions and Average Specific Cake Resistances by Filtration-Permeation Method

Fig. 3 shows the filtration-permeation results of 1 wt% calcium carbonate suspension. The straight line with slope at the left-hand side in the figure represents the filtration period. The average specific cake resistance is calculated conventionally with this slope and the initial mass fraction of solids in suspension by Eq. (4). The author [1986] termed the average specific cake resistance thus obtained as $\alpha_{av,f}$, and the value of $\alpha_{av,f}$ measured in this study was 8.4×10^{10} m/kg.

The suspension having such a dilute concentration did not form a clear solid blanket, but there formed supernatant as a result of sedimentation during filtration. The filtration process was finished when all of the suspension beneath the supernatant had been changed into cake. When filtration had been terminated, the supernatant still remained above the filter cake. After then the supernatant permeated through the filter cake. For the filtration of a dilute suspension with a relatively small amount, the time for filtration is not enough to have sufficient supernatant that would permit long permeation period. The permeation period was prolonged by adding particle-eliminated water to the filter cell. The author [1986] named this experimental method as filtration-permeation. The flow rate through the cake was constant during the permeation period, and this experimental result is presented in Fig. 3 with the horizontal points at the right hand side.

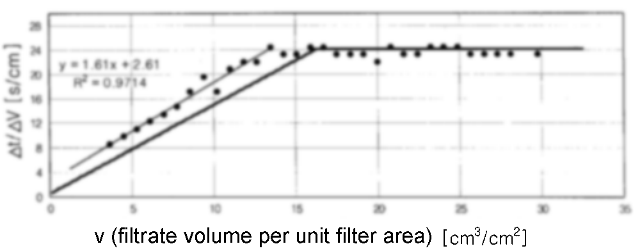


Fig. 3. Filtration-permeation result of 1 wt% $CaCO_3$ suspension with 0.45 μm MF membrane at 0.5 atm.

The average specific cake resistance can be calculated by Eq. (2) with the value of $\Delta t/\Delta v$ and the mass of solids in cake per unit filter area, W, and both of them do not change during the permeation period. The average specific cake resistance thus measured was named as $\alpha_{av,p}$, and was 7.4×10^{10} m/kg.

The value of R_m in Eq. (2) was determined by permeating the particle-eliminated water through the new filter medium which had no cake, and the value was 3.1×10^{9} m^{-1}.

The difference between $\alpha_{av,f}$ and $\alpha_{av,p}$ based on the $\alpha_{av,p}$ is 13 %. The $\alpha_{av,f}$ during filtration was measured with the existence of sedimentation, and the $\alpha_{av,p}$ during permeation was measured at the state when the sedimentation had been terminated. The accuracy of above two average specific cake resistances will be discussed later, but the difference of 13 % could be in the range of experimental error in filtration [Tiller and Shirato, 1964]. This means that the contribution of sedimentation on filtration of dilute suspension is not great. The author assumed that this effect is due to the fast progress of filtration resulting from dilute concentration of suspension.

To analyze the filtration process theoretically, the calculated result by conventional filtration theory with the average specific cake resistance during permeation, $\alpha_{av,p}$, is also shown in Fig. 3. The thick straight-line from the origin is the calculated result, and we assume that it represents the filtration process without the effect of sedimentation. Neither theory nor other values for calculation contained the factor concerning the effect of sedimentation. The slope of the straight line based on experimental data, which represents the filtration with sedimentation, is a little larger than that of the calculated line without sedimentation. The value of mass fraction of solids in suspension to be filtered, S, in Eq. (4) increases when sedimentation occurs during filtration, and the larger slope of the line in Fig. 3 results from the larger value of S.

With the above conceptions, another assumption is that the true average specific cake resistances during filtration and permeation must coincide when the effect of sedimentation is properly included in the filtration equation. Then the value of S can be calculated with the slope of the line based on experimental data and the average specific resistance without the effect of sedimentation, $\alpha_{av,p}$, by using Eq. (4). The α_{av} in the term $\mu\alpha_{av}/\Delta p[\rho S/1-(S/S_c)]$ of Eq. (4) is substituted with $\alpha_{av,p}$ which represents the average specific cake resistance without sedimentation. The value of the above term is the slope of the experimental data during filtration, 1.61 s/cm^2. The pressure drop for filtration was 0.507×10^{9} Pa, viscosity was 9.61×10^{-4} kg/ms, the density of liquid was 997.8 kg/m^3, and the mass fraction of solids in cake was 0.763. With these the mass fraction of solids in suspension during filtration with sedimentation was calculated as 0.0111. The mass fraction of solids in initial suspension was 0.1. So it was concluded that the mass fraction of solids in suspension was increased 11 % by the sedimentation during filtration.

The end point of filtration is the intersection point between the filtration line and the horizontal permeation line. The end point of filtration by the theoretical line in Fig. 3 is 16.0 cm^3/cm^2 in V, and that by the experimental line is 13.5 cm^3/cm^2. The latter is a little shorter than the former. It can be explained that a little concentrated suspension has been supplied to the cake by the effect of sedimentation ; the end point of filtration became a little earlier than the expected time calculated without sedimentation.

2. Effects of the Concentration of Suspension on the Filtration with Sedimentation

The filtration-permeation or filtration results of the suspensions, which had initial solid mass fraction of 0.04, 0.08 and 0.15, are shown in Fig. 4.

The three experimental results show almost the same trend, but the experimental values of $\Delta t/\Delta v$ at the permeation period vary with the concentrations of suspension. The theoretical calculated values assuming no sedimentation are shown with thick straight lines from the origin. Conclusions as below could be derived by analyzing the calculated and experimental results in the figure.

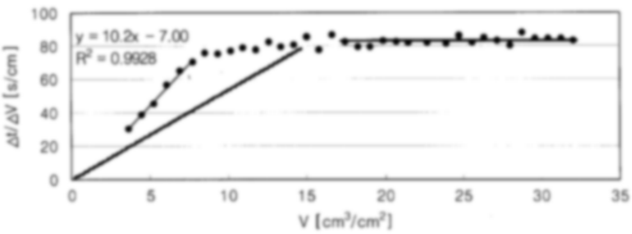
During the permeation period, the permeation velocities, $\Delta V/\Delta t$, remain at almost constant values in the above three cases. This means that the property of cake during permeation period does not change. Then the $\alpha_{av,p}$ can be calculated with the constant value of $\Delta t/\Delta V$ by Eq. (2). The conventional average specific resistance during filtration, $\alpha_{av,f}$, was also calculated with the slope in the figure during filtration, and the initial solid mass fraction in suspension by Eq. (4). The calculated results thus obtained are shown in Table 1.

In Fig. 4, the period of increasing $\Delta t/\Delta V$ is for filtration, and constant value of $\Delta t/\Delta V$ represents permeation. For all of the three above cases, the end of experimental filtrations which accompanied with sedimentation is faster than the end of theoretical filtrations. And the limit of experimental filtration is neither a function of initial mass fraction of solids in suspension nor a fixed value. As the thicker suspension sediments more slowly, it seems that the end point of filtration of thicker suspension would be prolonged. But, in fact, the time required for filtration of thicker suspension is also prolonged. There is no direct proportion between the end point and the suspension concentration. Nevertheless, the end point of filtration in Fig. 3 with 1 wt% suspension is much smaller than those in Fig. 4.

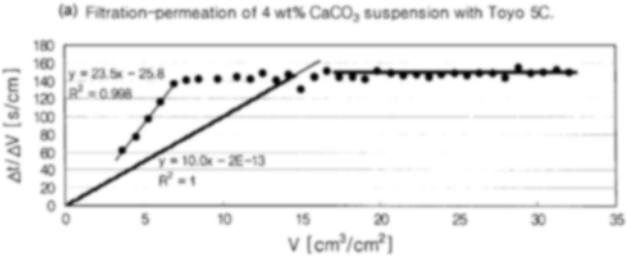
The interesting phenomenon in Fig. 4 is that all of the intercepts in the y-axis with the extrapolating line of experimental data have values below zero. There is no exact proportionality but the thicker suspension gives the smaller value of intercept. In conventional theory, the value of intercept means $\mu R_m/\Delta P$ in Eq. (4). Since the viscosity and pressure difference cannot have values below zero, the value of R_m must be below zero. But the minus value of R_m is not possible for filtration. We shall discuss this phenomenon a little later.

In Table 1, the average specific cake resistances obtained from filtration and permeation are presented. The conventional average specific resistances, $\alpha_{av,f}$'s, calculated from initial mass fraction of solids in suspension, which does not include the effect of sedimentation, give a difference of values up to 95 %.

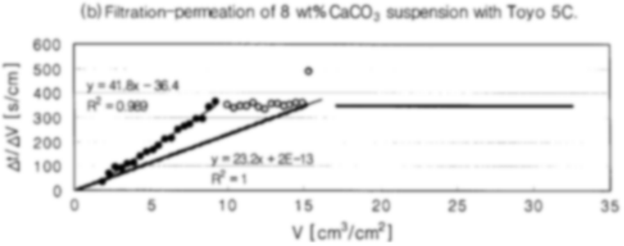
The difference between average specific cake resistances during permeation, $\alpha_{av,p}$'s, is maximum 7.5 %, but is almost null if the result of 1 % is excluded. With these permeation results, we can conclude that the property of a cake composed of the same material is identical even though the initial mass fraction of solids in suspension is different. The values of $\alpha_{av,p}$ are almost identical, but those of $\alpha_{av,f}$ which were measured during the formation of the same cakes show different values according to initial mass fraction of solids in suspension. With this experimental result it was assumed that there was an error or errors in the method of calculating $\alpha_{av,f}$. Until now, the explanation for the change of $\alpha_{av,f}$ according to initial mass fraction of solids in suspension was not found. Except for the author, almost all of the filtration engineers and researchers use the filtration period, i.e., $\alpha_{av,f}$, to measure the average specific cake resistance. Tiller [1995] only warned the danger of using $\alpha_{av,f}$. Tiller [1990] who could not explain the difference of the two kinds of average specific cake resistances concluded his paper as "Further analysis is required to account for the substantial



(a) Filtration-permeation of 4 wt% $CaCO_3$ suspension with Toyo 5C.



(b) Filtration-permeation of 8 wt% $CaCO_3$ suspension with Toyo 5C.



(c) Filtration of 15 wt% $CaCO_3$ suspension with 0.45 μm MF filter.

Fig. 4. Three experimental results of filtration with sedimentation.

Table 1. The average specific cake resistances of calcium carbonate suspensions for various initial concentrations

Initial conc. of $CaCO_3$ suspension	Average specific cake resistance by filtration, $\alpha_{av,f}$	Average specific cake resistance by permeation, $\alpha_{av,p}$
1 wt%	8.39×10^{10} m/kg	7.44×10^{10} m/kg
4 wt%	1.33×10^{11} m/kg	7.03×10^{10} m/kg
8 wt%	1.64×10^{11} m/kg	6.93×10^{10} m/kg
15 wt%	1.27×10^{11} m/kg	7.05×10^{10} m/kg

difference in the two sets of values." The two sets of values mean $\alpha_{av,f}$ and $\alpha_{av,p}$. He used the "computerized axial tomographic scanner (CATSCAN)" to analyse the phenomena during filtration with sedimentation, but he could not verify the difference between the two sets of average specific resistances. Sperry [1917] developed a technique which minimizes the influence of sedimentation during filtration by using the suspension already sedimented for two hours. But the effect of sedimentation during filtration could not be verified with this technique. In this study, a method to explain the difference between average specific cake resistances is proposed by the filtration of sediment as below.

3. Filtration of Sediment-Definition of the Sediment and Cake

Bockstal et al. [1985] and Tiller et al. [1995] made no difference between the cake and the sediment in their theories of filtration with sedimentation. They thought that the sediment is the cake. In this study, the filtration of sediment was performed to know the difference between cake and sediment. If sediment is cake, the filtration of sediment cannot be performed, and only the permeation result would be acquired.

The suspension of 17.6 wt% initial concentration was sedimented for 40 minutes in a filtrater cell before filtration. Sedimentation was completed in 30 minutes; after that, 10 additional minutes was added to ensure perfect sedimentation. The thickness of sediment thus obtained was 3.9 cm. The mass fraction of solids in sediment calculated with the thickness was 0.488 (48.8 wt%), and it is 2.8 times thicker than the initial mass fraction of solids in suspension.

The experimental result of the filtration-permeation of sediment is shown in Fig. 5. The formation of cake during the experiment was not visually observed, since the sediment and cake were all opaque and there was no visual difference. But the thickness of sediment decreased as time passed, and after a while the thickness did not change any more. The final thickness was 1.9 cm. The author explained this process as follows. At first, there was only sediment in the cell. The total thickness of sediment and cake decreased as filtration of sediment proceeded, because the solid content of cake is larger than that of sediment. When all of the sediment had changed into cake, the thickness did not change any more.

As the material which enters into cake was the sediment in this filtration, so the value of S in Eq. (4) was 0.488, i.e., the mass fraction of solids in sediment, instead of 0.176, that of solids in initial suspension. The average specific cake resistance

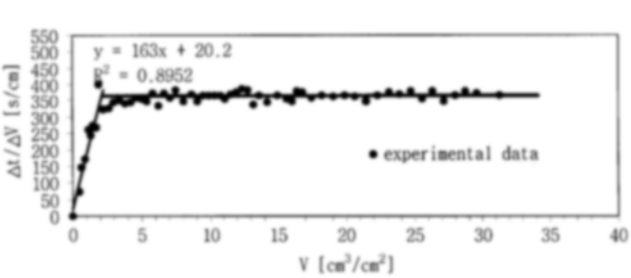


Fig. 5. Filtration-permeation of the sediment (48.8 wt% $CaCO_3$) prepared with a suspension of 17.6 wt% initial concentration.

during permeation, $\alpha_{av,p}$, is 7.28×10^{10} m/kg with the value of $\Delta t/\Delta v$ during permeation of 364 s/cm, the W of 28.6 kg/m^2, the pressure drop of filtration of 5×10^4 Pa, and viscosity of filtrate of 0.875 cP by the Eq. (2).

The average specific cake resistance during filtration, $\alpha_{av,f}$, by Eq. (4) is 7.23×10^{10} m/kg with a slope during filtration of 163.2 s/cm^2, measured cake solid content of 0.796, mass fraction of solids in sediment of 0.488, pressure drop of 5×10^4 Pa, and viscosity of filtrate 0.8049 cP. In fact, the above two average specific resistances coincide very well as a result of filtration-permeation.

As mentioned earlier, if the sediment was a cake, filtration did not exist and only permeation of the cake would be observed. But the filtration process is shown in Fig. 5 at the beginning of the experiment; the assumption of Bockstal et al. [1985] and Tiller et al. [1995] must be reexamined. With the above consideration, the cake can be redefined as follows. Cake is not only the accumulation of particles but the rearranged particles by the filtration pressure.

When the initial mass fraction of solids in suspension is used instead of sediment, the average specific cake resistance during filtration, $\alpha_{av,f}$, becomes 4.03×10^{11} m/kg, and this value is much different from the other values in Table 1. This value of $\alpha_{av,f}$ could not be true, because the initial suspension had not related to the formation of the cake in this experiment.

With same concept, the difference of $\alpha_{av,f}$ in Table 1 could be analyzed. The different values were caused by the initial concentrations which must be replaced with the true mass fraction of solids in suspension entering into the cake at each case.

The average specific resistances during permeation, $\alpha_{av,p}$'s, for the filtration of suspensions of initial mass fraction of solids from 0.01 to 0.488 show almost the same values within the limits of experimental error. It can be concluded that the cakes formed with various suspensions with initial concentration have almost the same value of average specific cake resistances.

Even if the above opinion would not be true, much caution is required for the use of $\alpha_{av,f}$ calculated from the slope of the $\Delta t/\Delta V$ vs. V and the initial concentration of suspension.

The same phenomenon is also shown for the filtration of the bentonite floc [Yim and Ben Aim, 1986; Yim, 1990].

4. The Values of R_m for the Filtration with Sedimentation

Conventionally, the value of R_m is calculated by Eq. (4) with the intercept of y-axis and the extrapolated line of the relation $\Delta t/\Delta V$ vs. V.

4-1. The Values of R_m in Case of the Filtration with Pre-sedimentation

When the diameter of pores of a filter medium is much larger than that of the particles in suspension, some particles pass through filter medium at the beginning period of filtration. In this case, the value of R_m by the extrapolation of $\Delta t/\Delta V$ vs. V is below zero, and this value cannot be treated as a true filter medium resistance. But in this study the diameter of the filter medium was not so large as to show this effect.

For analyzing the influence of sedimentation during the period from the moment of pouring suspension into the filter cell to the beginning of filtration, the filtration-permeation started after certain pre-sedimentation periods. The experimental results

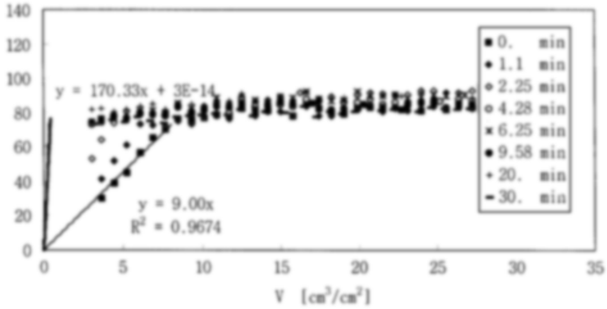


Fig. 6. Filtration-permeation results with a pre-sedimentation period for 4 wt% $CaCO_3$ suspension.

with initial concentration of 4 wt% for eight kinds of pre-sedimentation time are shown in Fig. 6. All experiments were performed at a pressure of 0.5 bar with a Toyo Advantec 5C filter as the filter medium.

The experimental result without pre-sedimentation, i.e., the result of filtration which had started immediately after placing suspension into the cell, shows a typical representative of filtration phenomena until a large value of v in the x-axis. And the extrapolated line of filtration data meets the y-axis near the origin. Sedimentation also occurred during this filtration procedure.

When filtration was started with 1.1 min pre-sedimentation time, in this moment the solid-blanket was at 1/8th of the initial suspension height, filtration was terminated a little earlier than the former case, but gave almost the same slope with the former case. Naturally, the intercept of the y-axis with the extrapolated line of filtration data in this case was much larger than that without pre-sedimentation. The larger intercept gives a larger value of R_m by Eq. (4).

Nearly the same result was obtained with the filtration of pre-sedimentation time of 2.25 min, i.e., the height of solid blanket was at 1/4th of the initial height. The filtration period was decreased compared to the former cases; only three data points were available during the filtration period. The slope with only three points has no meaning. The filtration behavior is not shown for the experiments with more pre-sedimentation time in Fig. 6.

In vacuum filtration, the pre-sedimentation time is not long because the filtration can be started just after pouring the suspension into the cell. But in the operation of pressure filtration, the time needed for sealing the filtration cell is at minimum one or two minutes. Pre-sedimentation occurs during this period, and the influences of the pre-sedimentation on filtration are shown in the above experimental results. One is the reduction of the filtration duration in the basis of v, x-axis, with increasing pre-sedimentation time, and this reduces the accuracy of the experiment. Another is the change of the value of R_m measured by conventional method with the pre-sedimentation time.

It cannot be explained by conventional theory that the medium resistance changes with the pre-sedimentation time. In this study, the filtration with pre-sedimentation is analyzed as follows with the conception of filtration of sediment. The sediment piles up upon a filter medium during pre-sedimentation time. When filtration begins, the filtration of sediment is performed. The mass fraction of solids entering cake, S, in this period is much greater than that of solids in initial suspension. The volume of sediment is small compared with total suspension; then the filtration of sediment ends at a small value of V. The calculated result of this process by the same method of former "filtration of sediment" is shown with the straight thick line from the origin in Fig. 6. After the filtration of sediment, filtration of the suspension is performed subsequently. The sedimentation is also accompanied during this period, so the value of S is not that of the initial period, 0.04. The beginning point of the filtration of suspension is determined from the amount of sediment. If all of the solids have been sedimented before the filtration begins, the filtration of suspension does not occur. Permeation of supernatant is continued with the filtration of sediment. The experimental results with long pre-sedimentation time in Fig. 6 show this phenomenon.

4-2. The Variation of R_m with the Concentration of Suspension

The intercepts with the extrapolation line of experimental filtration results and the y-axis in Fig. 4, which is the filtration-permeation results with varying initial concentrations of suspension, have values below zero (Naturally there was no pre-sedimentation). Filtration of suspensions of the thicker initial concentration shows the smaller value of intercept. The intercept of 4 wt% suspension was −7.0 s/cm, that of 8 wt% was −25.8 s/cm, and that of 15 wt% was −36.4 s/cm.

As mentioned above, the value of the intercept can be below zero when the pore size of the filter medium is much larger than the particle diameter. In this case some amount of particles pass through the filter medium and the particles are seen in filtrate. But this phenomenon is not possible in the above experiments, because the mean pore size of the filter medium Toyo 5C, i.e., 0.95 μm measured by permeation method [Yim et al., 1997], is much smaller than the mean volumetric particle size of 8.54 μm measured by Malvern Mastersizer.

The intercept in Fig. 1, which was the filtration of the initial suspension concentration of 0.01 wt%, is 2.61 s/cm, a positive value. And the intercept for the filtration of sediment, i.e., the filtration without sedimentation, is +20.2 s/cm. These two filtrations were performed with the same particle and filter medium. So the minus intercepts of the filtration of 4, 8, and 15 wt% suspensions must be explained with a different mechanism.

These three suspensions have relatively high initial concentrations which give slow velocity of sedimentation. The influence of sedimentation is relatively small at the beginning period of filtration, as the filtration velocity is fast and the sedimentation is slow with high suspension concentration. So the filtration proceeds along the line in Fig. 4 which was calculated without sedimentation. After a while, the mass of cake increases and the velocity of filtration becomes slow. The sedimentation influences the filtration process. The mass fraction of solids in suspension which enters into cake, S, becomes larger due to sedimentation.

The larger value of S gives a larger slope in the relation $\Delta t/\Delta V$ vs. V by Eq. (4). The slope is not so large at the beginning period during which experimental measurement is not possible, and then after the slope becomes a larger value. Without the data at the beginning period, the extrapolated line with the data of later period gives a value of the intercept below zero. The higher initial concentration of suspension gives slow velocity of sedimentation; the slow sedimentation prolongs the period with small value of slope. The prolongation of this period results in a smaller value of intercept by extrapolation of the data of later period. The experimental results in Fig. 4 prove this explanation.

The filtration of dilute suspension as 1 wt% in Fig. 3 did not show this phenomenon because the sedimentation effect is included from the beginning. The phenomenon was not observed for the filtration of **sediment** in Fig. 5, as there was no sedimentation.

The analysis of two special characteristics on R_m is not possible with other existing theories; it can be done by the above filtration with sediment and by the change of the concentration of suspension entering into a cake.

CONCLUSION

A series of filtration-permeations of calcium carbonate suspension were carried out with varying concentration of suspension at the same condition. There was 1.95 times difference in average specific cake resistance measured during filtration period with the initial mass fractions of solids in suspension. But the average specific resistances measured by the permeating velocity during permeation period coincided well within experimental error limits for wide range of initial concentrations.

It was assumed that the change of the values of average specific cake resistance by filtration with various initial concentrations was caused by the different sedimentation velocity which changed the mass fraction of solids entering into cake. To verify the assumption, filtration of sediment was conducted. As there exists no more sedimentation for a sediment, this experiment is a filtration without sedimentation. The average specific cake resistance by filtration period with the mass fraction of solids in sediment coincided well with that by permeation period, and also coincided with the above average specific resistances by permeation period with varying initial concentrations. It was concluded that the exact average specific cake resistance can be obtained with filtration data if the exact mass fraction of solids entering into cake is used. A new definition of cake is proposed by this experiment that sediment is not a cake and the rearranged particulate structure by filtration is a cake.

The effect of sedimentation of one or two minutes before filtration was verified with filtration with pre-sedimentation time. In this case, filtration of sediment is performed at the start, and then filtration with sedimentation is followed. If this phenomenon is ignored, a false increment in the value of R_m is obtained by conventional filtration theory which does not have a sedimentation concept.

An explanation of the minus values of R_m occasionally shown in the filtration of thick suspension was attempted with a comparison of the velocity of filtration and sedimentation.

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NOMENCLATURE

ΔP : filtration pressure [Pa]
R_m : resistance of filter medium [m^{-1}]
S : mass fraction of solids in suspension which enters into a filter cake [-]
S_c : mass fraction of solids in cake [-]
t : time for filtration [s]
V : filtrate volume per unit filter area [m^3/m^2]
W : dry cake mass per unit filter area [kg/m^3]

Greek Letters

α_{av} : average specific cake resistance [m/kg]
$\alpha_{av,f}$: average specific cake resistance by filtration [m/kg]
$\alpha_{av,p}$: average specific cake resistance by permeation [m/kg]
μ : viscosity of filtrate [kg/ms]
ρ : density of filtrate [kg/m^3]

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